3 Surds and Percentages

M11CH3

3.1. SURDS

Definition: Let 'a' be a rational number and 'n' be a positive integer such that $\sqrt[n]{a}$ is irrational number, then $\sqrt[n]{a}$ or $a^{\frac{1}{n}}$ is called a **surd** or radical of order n. The rational number 'a' is called the **radicand**. The symbol $\sqrt[n]{\ }$ is called the **radical sign**.

In simple words, the roots ($\sqrt[n]{\ }$) that cannot be further simplified into whole numbers or integers and give irrational numbers (a decimal which neither repeats nor terminates) are called surds. Surds are irrational numbers.

The examples of surds are $\sqrt{2}, \sqrt{3}, \sqrt[3]{6}, \sqrt[4]{4}, \sqrt[9]{2}$, etc., as these values cannot be further simplified. If we further simply them, we get decimal values, such as:

$\sqrt{2}$ = 1.414213562..., $\sqrt{3}$ = 1.732050808..., $\sqrt[3]{6}$ = 1.817120593..., etc.

If a calculation for a surd required in questions, irrational numbers must be left in the surd form.

A surd of order 2 is called a quadratic surd. In this chapter, we shall study only quadratic surds. Thus, $\sqrt[2]{a}$ or simply $\sqrt{a}$ is a quadratic surd where the radicand 'a' must be positive rational number which is not a perfect square. The sign $\sqrt{\ \ }$ means the positive square root.

For example, $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{10}, \sqrt{11}, \sqrt{13}$, ..., etc. are quadratic surds. Since negative real number do not have square roots i.e., $\sqrt{-3}$ and $\sqrt{-9}$ are quadratic surds. Also, $\sqrt{4}$ = 2, $\sqrt{9}$ = 3, $\sqrt{16}$ = 4, $\sqrt{25}$ = 5, $\sqrt{36}$ = 6, ..., etc. is not irrational, therefore, $\sqrt{4}, \sqrt{9}, \sqrt{16}, \sqrt{25}, \sqrt{36}$, ..., etc. are not a quadratic surd.

Notes:
1. The square roots of prime numbers are all surds.
2. The square roots of any number other than square numbers are surds.

3.2. SIMPLIFYING SURDS

Properties of Surds

1. $\sqrt{a^2} = a$

For example: $\sqrt{5^2} = 5$

2. $\sqrt{a} \times \sqrt{a} = (\sqrt{a})^2 = a$

For example: $\left(\sqrt{\frac{3}{5}}\right)^2 = \frac{3}{5}$

3. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ and $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

For example: $\sqrt{8} \times \sqrt{2} = \sqrt{8 \times 2} = \sqrt{16} = \sqrt{4^2} = 4$

$\sqrt{15} = \sqrt{5 \times 3} = \sqrt{5} \times \sqrt{3}$

Thus, the square root of a product is the product square roots.

4. $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

For example: $\frac{\sqrt{15}}{\sqrt{10}} = \sqrt{\frac{15}{10}} = \sqrt{\frac{3}{2}}$

$\sqrt{\frac{7}{10}} = \frac{\sqrt{7}}{\sqrt{10}}$

Thus, the square root of a quotient is the quotient of square roots.

5. $a\sqrt{b} \times \sqrt{c} = a\sqrt{bc}$

For example: $3\sqrt{2} \times \sqrt{3} = 3\sqrt{2 \times 3} = 3\sqrt{6}$

6. $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$

For example: $2\sqrt{3} \times 5\sqrt{4} = (2 \times 5)\sqrt{3 \times 4} = 10\sqrt{12}$

7. $a \times \sqrt{b} = a\sqrt{b}$

For example: $3 \times \sqrt{5} = 3\sqrt{5}$

8. $a \times b\sqrt{c} = ab\sqrt{c}$

For example: $3 \times 4\sqrt{2} = (3 \times 4)\sqrt{2} = 12\sqrt{2}$

Simplifying Surds by Reducing to Basic Form

Now, we shall use these properties or facts to simplify surds.

A quadratic surd $\sqrt{a}$ is said to be simplified if its radicand 'a' has no factor which is a perfect square.

In other words, a quadratic surd $\sqrt{a}$ is said to simplified if its radicand 'a' cannot be reduced further, then it is said to be in the **basic form**.

For example, $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \ldots$, etc. are in basic forms and cannot be reduced or simplified any further.

To simplify a quadratic surd $\sqrt{a}$ where 'a' is a positive integer, write $a = m^2 \times n$ where m, n are positive integers and n is not a perfect square, i.e., m^2 is the largest perfect square factor of a.

Then, $\sqrt{a} = \sqrt{m^2 \times n} = \sqrt{m^2} \times \sqrt{n} = m\sqrt{n}$

For example:

(i) $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

(ii) $\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

(iii) $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$

(iv) $\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$

(v) $\dfrac{\sqrt{12}}{\sqrt{3}} = \sqrt{\dfrac{12}{3}} = \sqrt{4} = 2$

(vi) $\sqrt{\dfrac{5}{9}} = \dfrac{\sqrt{5}}{\sqrt{9}} = \dfrac{\sqrt{5}}{3}$

Note:

Keep in mind the following perfect squares while simplifying surds:

$\sqrt{4} = 2$	$\sqrt{64} = 8$	$\sqrt{196} = 14$	$\sqrt{400} = 20$
$\sqrt{9} = 3$	$\sqrt{81} = 9$	$\sqrt{225} = 15$	$\sqrt{441} = 21$
$\sqrt{16} = 4$	$\sqrt{100} = 10$	$\sqrt{256} = 16$	$\sqrt{484} = 22$
$\sqrt{25} = 5$	$\sqrt{121} = 11$	$\sqrt{289} = 17$	$\sqrt{529} = 23$
$\sqrt{36} = 6$	$\sqrt{144} = 12$	$\sqrt{324} = 18$	$\sqrt{576} = 24$
$\sqrt{49} = 7$	$\sqrt{169} = 13$	$\sqrt{361} = 19$	$\sqrt{625} = 25$

Example 1: *Simplify the following surds:*

(i) $\sqrt{28}$ (ii) $\sqrt{54}$ (iii) $\sqrt{180}$

(iv) $\sqrt{243}$ (v) $\sqrt{432}$ (vi) $\sqrt{\frac{75}{450}}$

Solution:

(i) $\sqrt{28} = \sqrt{4 \times 7} = \sqrt{4} \times \sqrt{7} = 2\sqrt{7}$

(ii) $\sqrt{54} = \sqrt{9 \times 6} = \sqrt{9} \times \sqrt{6} = 3\sqrt{6}$

(iii) $\sqrt{180} = \sqrt{36 \times 5} = \sqrt{36} \times \sqrt{5} = 6\sqrt{5}$

(iv) $\sqrt{243} = \sqrt{81 \times 3} = \sqrt{81} \times \sqrt{3} = 9\sqrt{3}$

(v) $\sqrt{432} = \sqrt{144 \times 3} = \sqrt{144 \times 3} = 12\sqrt{3}$

Alternatively,

$\sqrt{432} = \sqrt{3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2}$ (After factorisation)

$= \sqrt{\mathbf{3 \times 3} \times 3 \times \mathbf{2 \times 2} \times \mathbf{2 \times 2}}$ (Pairing)

$= (3 \times 2 \times 2)\sqrt{3}$

$= 12\sqrt{3}$

(vi) $\sqrt{\frac{75}{450}} = \sqrt{\frac{15}{90}} = \sqrt{\frac{1}{6}} = \frac{\sqrt{1}}{\sqrt{6}} = \frac{1}{\sqrt{6}}$

Example 2: *State whether the following are surds or not with reasons :*

(i) $\sqrt{27}\times\sqrt{3}$ (ii) $\sqrt{5}\times\sqrt{10}$

(iii) $\sqrt{6}\times\sqrt{8}$ (iv) $\sqrt{16}\times\sqrt{4}$

Solution:

(i) $\sqrt{27}\times\sqrt{3}=\sqrt{27\times3}=\sqrt{81}=9$, which is a rational number. Hence, it is not a surd.

(ii) $\sqrt{5}\times\sqrt{10}=\sqrt{5\times10}=\sqrt{50}=\sqrt{25\times2}=\sqrt{25}\times\sqrt{2}=5\sqrt{2}$, which is an irrational number. Hence, it is a surd.

(iii) $\sqrt{6}\times\sqrt{8}=\sqrt{6\times8}=\sqrt{48}=\sqrt{16\times3}=\sqrt{16}\times\sqrt{3}=4\sqrt{3}$, which is a irrational number. Hence, it is a surd.

(iv) $\sqrt{16}\times\sqrt{4}=\sqrt{16\times4}=\sqrt{64}=8$, which is a rational number. Hence, it is not a surd.

EXERCISE 3.1

1. Simplify the following:

(i) $\sqrt{20}$ (ii) $\sqrt{27}$ (iii) $\sqrt{32}$

(iv) $\sqrt{45}$ (v) $\sqrt{80}$ (vi) $\sqrt{75}$

(vii) $\sqrt{200}$ (viii) $\sqrt{147}$ (ix) $\sqrt{128}$

(x) $\sqrt{96}$ (xi) $\sqrt{7}\times\sqrt{24}$ (xii) $\sqrt{2}\times\sqrt{3}\times\sqrt{8}$

(xiii) $\dfrac{\sqrt{60}}{\sqrt{15}}$ (xiv) $\dfrac{\sqrt{20}}{\sqrt{8}}$ (xv) $\sqrt{\dfrac{80}{49}}$

(xvi) $\sqrt{\dfrac{98}{162}}$ (xvii) $\sqrt{0.09}$ (xviii) $\sqrt{0.0064}$

2. State whether the following are surds or not with reasons:

(i) $\sqrt{100}\times\sqrt{2}$ (ii) $\sqrt{125}\times\sqrt{5}$ (iii) $\sqrt{120}\times\sqrt{45}$

(iv) $\sqrt{18}\times\sqrt{50}$ (v) $\sqrt{24}\times\sqrt{27}$ (vi) $\sqrt{200}\times\sqrt{125}$

Addition and Subtraction of Surds

Quadratic surds having the same radicand or same form can be added and subtracted. Such radicals are called **like radicals**.

Notes:

1. Before addition or substraction, reduce the surds into their basic form if they are not.
2. $\sqrt{a}+\sqrt{b} \neq \sqrt{a+b}$

   For example: $\sqrt{7}+\sqrt{2} \neq \sqrt{7+2}$

   i.e. $\sqrt{7}+\sqrt{2} \neq \sqrt{9}$

   or $\sqrt{7}+\sqrt{2} \neq 3$
3. $\sqrt{a}-\sqrt{b} \neq \sqrt{a-b}$

   For example: $\sqrt{7}-\sqrt{3} \neq \sqrt{7-3}$

   i.e. $\sqrt{7}-\sqrt{3} \neq \sqrt{4}$

   or $\sqrt{7}-\sqrt{3} \neq 2$

Example 3: *Simplify the following:*

(i) $5\sqrt{3}+2\sqrt{5}$ (ii) $12\sqrt{5}-7\sqrt{5}$

(iii) $3\sqrt{3}+7\sqrt{3}-12\sqrt{3}$

Solution:

(i) $5\sqrt{3}+2\sqrt{3} = (5+2)\sqrt{3} = 7\sqrt{3}$

(ii) $12\sqrt{5}-7\sqrt{5} = (12-7)\sqrt{5} = 5\sqrt{5}$

(iii) $3\sqrt{3}+7\sqrt{3}-12\sqrt{3} = 10\sqrt{3}-12\sqrt{3} = (10-12)\sqrt{3} = -2\sqrt{3}$

Example 4: *Evaluate the following:*

(i) $\sqrt{18}+\sqrt{8}$ (ii) $7\sqrt{48}-4\sqrt{27}$

(iii) $12\sqrt{45}-25\sqrt{20}$ (iv) $3\sqrt{500}-10\sqrt{125}+5\sqrt{80}$

(v) $5\sqrt{32}+\frac{2}{3}\sqrt{162}-\frac{4}{5}\sqrt{200}$

Solution:

(i) $\sqrt{18}+\sqrt{8}$

[Here 18 and 8 can be split into two factors such that one is perfect square.]

$$= \sqrt{9\times 2}+\sqrt{4\times 2} = \sqrt{9}\times\sqrt{2}+\sqrt{4}\times\sqrt{2} = 3\sqrt{2}+2\sqrt{2}$$

$$= (3+2)\sqrt{2} = 5\sqrt{2}$$

(ii) $7\sqrt{48}-4\sqrt{27} = 7\sqrt{16\times 3}-4\sqrt{9\times 3}$

$$= 7(4\sqrt{3})-4\times(3\sqrt{3}) = 28\sqrt{3}-12\sqrt{3}$$

$$= (28-12)\sqrt{3} = 16\sqrt{3}$$

(iii) $12\sqrt{45}-25\sqrt{20} = 12\sqrt{9\times 5}-25\sqrt{4\times 5} = 12(3\sqrt{5})-25(2\sqrt{5})$

$$= 36\sqrt{5}-50\sqrt{2} = (36-50)\sqrt{2} = -14\sqrt{2}$$

(iv) $3\sqrt{500}-10\sqrt{125}+5\sqrt{80} = 3\sqrt{100\times 5}-10\sqrt{25\times 5}+5\sqrt{16\times 5}$

$$= 3(10\sqrt{5})-10(5\sqrt{5})+5(4\sqrt{5})$$

$$= 30\sqrt{5}-50\sqrt{5}+20\sqrt{5} = 0$$

(v) $5\sqrt{32}+\frac{2}{3}\sqrt{162}-\frac{4}{5}\sqrt{200} = 5\sqrt{16\times 2}+\frac{2}{3}\sqrt{81\times 2}-\frac{4}{5}\sqrt{100\times 2}$

$$= 5(4\sqrt{2})+\frac{2}{3}(9\sqrt{2})-\frac{4}{5}(10\sqrt{2}) = 20\sqrt{2}+6\sqrt{2}-8\sqrt{2} = 18\sqrt{2}$$

Example 5: *Find the value of p, if* $2\sqrt{27}-\sqrt{75}-\sqrt{147} = p\sqrt{3}$

Solution: We have,

$$3\sqrt{27}-\sqrt{75}-\sqrt{147} = p\sqrt{3}$$

$$\Rightarrow 3\sqrt{9\times 3}-\sqrt{25\times 3}-\sqrt{49\times 3} = p\sqrt{3}$$

$$\Rightarrow 9\sqrt{3}-5\sqrt{3}-7\sqrt{3} = p\sqrt{3}$$

$$\Rightarrow -3\sqrt{3} = p\sqrt{3}$$

Comparing both sides, we get $p = -3$

EXERCISE 3.2

1. Carry out the indicated operations:

(i) $7\sqrt{2}+5\sqrt{2}$ (ii) $15\sqrt{3}+6\sqrt{3}$

(iii) $\frac{3}{2}\sqrt{5}+\frac{1}{2}\sqrt{5}$ (iv) $9\sqrt{6}-5\sqrt{6}$

(v) $37\sqrt{11}-25\sqrt{11}$ (vi) $27\sqrt{10}-12\sqrt{10}-5\sqrt{10}$

(vii) $8\sqrt{8}-5\sqrt{18}$ (viii) $7\sqrt{27}-2\sqrt{12}$

(ix) $12\sqrt{7}+5\sqrt{7}-9\sqrt{7}$ (x) $3\sqrt{50}+5\sqrt{32}-7\sqrt{18}$

2. Simplify the following :

(i) $\sqrt{32}+\sqrt{72}-\sqrt{162}$

(ii) $\sqrt{27}-\sqrt{75}+\sqrt{147}$

(iii) $5\sqrt{20}-3\sqrt{320}+\frac{3}{5}\sqrt{500}$

(iv) $\sqrt{24}-\sqrt{54}-\sqrt{216}+\sqrt{294}$

3. Find the values of p if

(i) $\sqrt{50}-\sqrt{18}+\sqrt{32}+\sqrt{128} = p\sqrt{2}$

(ii) $2\sqrt{54}+\sqrt{150}-\sqrt{384} = p\sqrt{6}$

(iii) $3\sqrt{180}+2\sqrt{245}-5\sqrt{405} = p\sqrt{5}$

3.3. PRODUCT AND QUOTIENTS OF SURDS

Product/Multiplication of Surds

The product/multiplication of surds in brackets follows the rules/ properties of surds. In case of product or multiplication of two surds, there is generally no need to simplify the surds into basic forms.

Note: Sometimes, two quadratic surds follow the rule of difference of two squares. The rule of difference of two square is :

$$(a + b)(a - b) = a^2 - b^2$$

For example, $(3+\sqrt{2})(3-\sqrt{2}) = 3^2-(\sqrt{2})^2 = 9-2=7$

Example 6: *Simplify each of the following :*

(i) $(2+\sqrt{3})(2-\sqrt{3})$ (ii) $(5-\sqrt{3})(5+\sqrt{3})$

(iii) $(2-\sqrt{5})(6+\sqrt{5})$ (iv) $(2+\sqrt{2})(8+\sqrt{8})$

(v) $5\sqrt{3}\times\sqrt{3}$ (vi) $2\sqrt{12}\times 7\sqrt{20}\times\sqrt{32}$

(vii) $(\sqrt{3})^5\times(\sqrt{75})$ (viii) $3\sqrt{2}(3-2\sqrt{2})+4\sqrt{3}(2+\sqrt{3})$

Solution:

(i) $(2+\sqrt{3})(2-\sqrt{3}) = 2^2-(\sqrt{3})^2 = 4-3 = 1$

Alternatively,

$$(2+\sqrt{3})(2-\sqrt{3}) = 2(2-\sqrt{3})+\sqrt{3}(2-\sqrt{3}) \text{ (Expanding)}$$
$$= 4-2\sqrt{3}+2\sqrt{3}-(\sqrt{3})^2$$
$$= 4-2\sqrt{3}+2\sqrt{3}-3$$
(Adding like terms)
$$= 1$$

(ii) $(5-\sqrt{3})(5+\sqrt{3}) = 5^2-(\sqrt{3})^2 = 25-3 = 22$

(iii) $(2-\sqrt{5})(6+\sqrt{5}) = 2(6+\sqrt{5})-\sqrt{5}(6+\sqrt{5})$ (Expanding)
$$= 12+2\sqrt{5}-6\sqrt{5}-(\sqrt{5})^2$$
$$= 12+2\sqrt{5}-6\sqrt{5}-5$$
$$= (12-5)+(2-6)\sqrt{5}$$
(Adding like terms)
$$= 7-4\sqrt{5}$$

(iv) $(2+\sqrt{2})(8+\sqrt{8}) = (2+\sqrt{2})(8+\sqrt{4\times 2})$
$$= (2+\sqrt{2})(8+2\sqrt{2})$$
$$= 16+4\sqrt{2}+8\sqrt{2}+2(\sqrt{2})^2$$
(Expanding)
$$= 16+4\sqrt{2}+8\sqrt{2}+4$$
$$= 20+12\sqrt{2} \quad \text{(Adding like terms)}$$

(v) $5\sqrt{3}\times\sqrt{3} = 5\times(\sqrt{3\times 3}) = 5\sqrt{9} = 5\times 3 = 15$

(vi) $2\sqrt{12}\times 7\sqrt{20}\times\sqrt{32} = 2(\sqrt{4\times 3})\times 7(\sqrt{4\times 5})\times(\sqrt{16\times 2})$

$= 2(2\sqrt{3})\times 7(2\sqrt{5})\times 4\sqrt{2}$

$= 4\sqrt{3}\times 14\sqrt{5}\times 4\sqrt{2}$

$= (4\times 14\times 4)\sqrt{3\times 5\times 2}$

$= 224\sqrt{30}$

(vii) $(\sqrt{3})^5\times(\sqrt{75}) = (\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3})\times\sqrt{25\times 3}$

$= (9\sqrt{3})\times(5\sqrt{3}) = (9\times 5)\sqrt{3\times 3}$

$= 45\times 3 = 135$

(viii) $3\sqrt{2}(3-2\sqrt{2})+4\sqrt{3}(2+\sqrt{3}) = 9\sqrt{2}-12+8\sqrt{3}+12 = 9\sqrt{2}+8\sqrt{3}$

Example 7: Simplify $\sqrt{1500}+\sqrt{3}(\sqrt{3}+5\sqrt{5})+2\sqrt{5}(\sqrt{3}-\sqrt{5})$ *in the form* $p + q\sqrt{15}$, *where p and q are integers.*

Solution: $\sqrt{1500}+\sqrt{3}(\sqrt{3}+5\sqrt{5})+2\sqrt{5}(\sqrt{3}-\sqrt{5})$

$= \sqrt{100\times 15}+(\sqrt{3})^2+5\sqrt{3\times 15}+2\sqrt{5\times 3}-2(\sqrt{5})^2$

$= 10\sqrt{15}+3+5\sqrt{15}+2\sqrt{15}-10$

$= -7+17\sqrt{15}$

Example 8: *If* $(2-3\sqrt{3})(3+2\sqrt{3}) = p+q\sqrt{3}$ *then, find the values of p and q.*

Solution: $(2-3\sqrt{3})(3+2\sqrt{3}) = p+q\sqrt{3}$

$\Rightarrow 6+4\sqrt{3}-9\sqrt{3}-6(3) = p+q\sqrt{3}$

$\Rightarrow 6+4\sqrt{3}-9\sqrt{3}-18 = p+q\sqrt{3}$

$\Rightarrow -12-5\sqrt{3} = p+q\sqrt{3}$

Comparing both sides, we get

$p = -12$ and $q = -5$

EXERCISE 3.3

1. Simplify :

(i) $\sqrt{3}(\sqrt{6}+\sqrt{24})$ (ii) $4\sqrt{27}\times 5\sqrt{28}$

(iii) $(1-\sqrt{3})(1+3\sqrt{3})$ (iv) $(3+2\sqrt{5})(3\sqrt{5}-2)$

(v) $(5+2\sqrt{6})(5-2\sqrt{6})$ (vi) $(6-2\sqrt{3})^2$

(vii) $2\sqrt{3}(2-\sqrt{3})+3\sqrt{2}(\sqrt{2}+1)$ (viii) $(7\sqrt{3}+4\sqrt{7})(2\sqrt{7}-\sqrt{3})$

2. Find the values of p and q if

(i) $(3\sqrt{3}-2)(3\sqrt{3}+2) = p + q\sqrt{3}$

(ii) $(3+4\sqrt{3})(2-2\sqrt{3}) = p - q\sqrt{3}$

Quotient or Division of Surds

The quotient of surd is result obtained by dividing one surd by another. In division of surds, we need to divide a given surd by another surd, the quotient of first expressed as a fraction. Then by rationalising the denominator (if the denominator is a surd), the required quotient is obtained with a rational denominator. For this, both the numerator and the denominator are multiplied by a particular rationalising factor.

Rationalising a Surd with Monomial Denominator

Rationalising surds (or rationalising a denominator) is a process of making the denominator of fractional surds from an irrational number to a rational number or whole number.

Surds of the form $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \ldots$ etc. can be rationalised by multiplying both the numerator and the denominator to rationalize the surd of the form $\frac{a}{\sqrt{b}}$, where $\sqrt{b}$ is the simplest form is given by:

$$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}}\times\frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{(\sqrt{b})^2} = \frac{a\sqrt{b}}{b} \qquad (\because (\sqrt{b})^2 = b)$$

This operation shifts the radical sign ($\sqrt{\ \ }$) from denominator to the numerator, thus making the free from radical sign. The factor $\sqrt{b}$ which rationalizes the denominator is called the rationalising factor.

Example 9: *Rationalize the denominator in each of the following.*

(i) $\dfrac{3}{\sqrt{2}}$ (ii) $\dfrac{7}{\sqrt{18}}$ (iii) $\dfrac{3\sqrt{5}}{\sqrt{3}}$ (iv) $\dfrac{\sqrt{3}}{\sqrt{72}}$

Solution:

(i) $$\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{(\sqrt{2})^2} = \frac{3\sqrt{2}}{2}$$

(ii) $$\frac{7}{\sqrt{18}} = \frac{7}{\sqrt{9\times 2}} = \frac{7}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{3(\sqrt{2})^2} = \frac{7\sqrt{2}}{3\times 2} = \frac{7\sqrt{2}}{6}$$

(iii) $$\frac{3\sqrt{5}}{\sqrt{3}} = \frac{3\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{15}}{(\sqrt{3})^2} = \frac{3\sqrt{15}}{3} = \sqrt{15}$$

(iv) $$\frac{\sqrt{3}}{\sqrt{72}} = \frac{\sqrt{3}}{\sqrt{36\times 2}} = \frac{\sqrt{3}}{6\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{6(\sqrt{2})^2} = \frac{\sqrt{6}}{6\times 2} = \frac{\sqrt{6}}{12}$$

Example 10: *Simplify the following:*

(i) $\sqrt{99} + \dfrac{2}{\sqrt{11}}$ (ii) $\dfrac{1}{\sqrt{2}}\left(\sqrt{18} - \sqrt{32} + \dfrac{3}{\sqrt{2}}\right)$

(iii) $\dfrac{1}{\sqrt{2}} + \dfrac{1}{3\sqrt{2}} + \dfrac{\sqrt{2}}{3}$ (iv) $\left(\sqrt{5} - \dfrac{3}{\sqrt{5}}\right)\left(\dfrac{4}{\sqrt{5}} + 6\right)$

(v) $\sqrt{7}\left(\sqrt{63} + \dfrac{32}{\sqrt{112}}\right)$ (vi) $\sqrt{5}\left(3\sqrt{45} - \dfrac{28}{\sqrt{245}}\right)$

Solution:

(i) $$\sqrt{99} + \frac{2}{\sqrt{11}} = \sqrt{9\times 11} + \frac{2}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}}$$

$$= 3\sqrt{11} + \frac{2\sqrt{11}}{(\sqrt{11})^2} = 3\sqrt{11} + \frac{2\sqrt{11}}{11}$$

$$= 3\sqrt{11} + \frac{2}{11}\sqrt{11} = \left(3 + \frac{2}{11}\right)\sqrt{11}$$

$$= \frac{35}{11}\sqrt{11}$$

(ii) $$\frac{1}{\sqrt{2}}\left(\sqrt{18} - \sqrt{32} + \frac{3}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}\left(\sqrt{9\times 2} - \sqrt{16\times 2} + \frac{3}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}\left(3\sqrt{2} - 4\sqrt{2} + \frac{3}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}\left(-\sqrt{2} + \frac{3}{\sqrt{2}}\right) = -\frac{\sqrt{2}}{\sqrt{2}} + \frac{3}{(\sqrt{2})^2}$$

$$= -1 + \frac{3}{2} = \frac{1}{2}$$

(iii) $$\frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{2}} + \frac{\sqrt{2}}{3} = \frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{2}}{3}$$

$$= \frac{\sqrt{2}}{(\sqrt{2})^2} + \frac{\sqrt{2}}{3(\sqrt{2})^2} + \frac{\sqrt{2}}{3}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{6} + \frac{\sqrt{2}}{3} = \left(\frac{1}{2} + \frac{1}{6} + \frac{1}{3}\right)\sqrt{2}$$

$$= \left(\frac{3+1+2}{6}\right)\sqrt{2} = \sqrt{2}$$

(iv) $$\left(\sqrt{5} - \frac{3}{\sqrt{5}}\right)\left(\frac{4}{\sqrt{5}} + 6\right) = \frac{4\sqrt{5}}{\sqrt{5}} + 6\sqrt{5} - \frac{12}{(\sqrt{5})^2} - \frac{18}{\sqrt{5}}$$

$$= 4 + 6\sqrt{5} - \frac{12}{5} - \frac{18}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$$

$$= \left(4 - \frac{12}{5}\right) + 6\sqrt{5} - \frac{18\sqrt{5}}{5}$$

$$= \frac{8}{5} + \left(6 - \frac{18}{5}\right)\sqrt{5} = \frac{8}{5} + \frac{12}{5}\sqrt{5}$$

(v) $$\sqrt{7}\left(\sqrt{63}+\frac{32}{\sqrt{112}}\right) = \sqrt{7}\left(\sqrt{9\times 7}+\frac{32}{\sqrt{16\times 7}}\right)$$

$$= \sqrt{7}\left(3\sqrt{7}+\frac{32}{4\sqrt{7}}\right)$$

$$= 3(\sqrt{7})^2+\frac{32\sqrt{7}}{4\sqrt{7}} = 21+8=28$$

(vi) $$\sqrt{5}\left(3\sqrt{45}-\frac{28}{\sqrt{245}}\right) = \sqrt{5}\left(3\sqrt{9\times 5}-\frac{28}{\sqrt{49\times 5}}\right)$$

$$= \sqrt{5}\left(3(3\sqrt{5})-\frac{28}{7\sqrt{5}}\right)$$

$$= \sqrt{5}\left(9\sqrt{5}-\frac{4}{\sqrt{5}}\right) = 9(\sqrt{5})^2-\frac{4\sqrt{5}}{\sqrt{5}}$$

$$= 45 - 4 = 41$$

Example 11: *Let * be a binary operation defined by* $a * b = a^2 + b^2 - 2ab$, *where a and b are non-zero real numbers, then find the values of the following :*

(i) $\sqrt{3} * \sqrt{12}$ (ii) $\sqrt{8} * \sqrt{200}$ (iii) $\sqrt{5} * \frac{1}{\sqrt{75}}$

Solution: We have, $a * b = a^2 + b^2 - 2ab$

(i) $$\sqrt{3} * \sqrt{12} = (\sqrt{3})^2+(\sqrt{12})^2-2(\sqrt{3})(\sqrt{12})$$

$$= 3 + 12 - 2\sqrt{36} = 3 + 12 - 2(6) = 3 + 12 - 12$$

$$= 3$$

(ii) $$\sqrt{8} * \sqrt{200} = (\sqrt{8})^2+(\sqrt{200})^2-2(\sqrt{8})(\sqrt{200})$$

$$= 8 + 200 - 2\sqrt{1600} = 208 - 2\sqrt{16\times 100}$$

$$= 208 - 2(\sqrt{16}\times\sqrt{100}) = 208 - 2(4\times 10)$$

$$= 208 - 80 = 128$$

(iii) $$\sqrt{5} * \frac{1}{\sqrt{75}} = (\sqrt{5})^2 + \left(\frac{1}{\sqrt{75}}\right)^2 - 2(\sqrt{3})\left(\frac{1}{\sqrt{75}}\right)$$

$$= 5 + \frac{1}{75} - \frac{2\sqrt{3}}{\sqrt{75}} = 5 + \frac{1}{75} - 2\sqrt{\frac{3}{75}}$$

$$= 5 + \frac{1}{75} + 2\sqrt{\frac{1}{25}} = 5 + \frac{1}{75} - \frac{2}{5}$$

$$= \frac{375 + 1 - 30}{75} = \frac{346}{75}$$

EXERCISE 3.4

1. Rationalize the denominator in each of the following :

(i) $\frac{1}{\sqrt{5}}$ (ii) $\frac{3}{\sqrt{7}}$ (iii) $\frac{6\sqrt{2}}{\sqrt{3}}$

(iv) $\frac{5}{\sqrt{8}}$ (v) $\frac{2}{\sqrt{18}}$ (vi) $\frac{7}{\sqrt{45}}$

(vii) $\frac{3\sqrt{5}}{\sqrt{50}}$ (viii) $\frac{9}{\sqrt{80}}$

2. Simplify the following :

(i) $\sqrt{63} + \frac{14}{\sqrt{7}}$ (ii) $\frac{1}{\sqrt{3}}(\sqrt{6} + \sqrt{12})$

(iii) $(1 - \sqrt{3})\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right)$ (iv) $(5 + 2\sqrt{5})\left(3 - \frac{2}{\sqrt{5}}\right)$

(v) $(7 + 3\sqrt{7})\left(2 - \frac{3}{\sqrt{7}}\right)$

3. If $p * q = p^2 + q^2 + 2\,pq$, find

(i) $\sqrt{3} * \sqrt{12}$ (ii) $\sqrt{5} * \sqrt{20}$ (iii) $\sqrt{3} * \frac{1}{\sqrt{12}}$

3.4. COMPOUND INTEREST IN RELATION TO SIMPLE INTEREST

Simple Interest

In our daily life, we have seen that when some money is borrowed, extra money is paid to the lender for the use of the borrowed money for a given period of time at a given rate.

The **extra money** paid is called interest.

Also, interest is extra money paid by institutions like banks on money deposited (kept) with them.

When interest is calculated yearly, half yearly or quarterly, but not added to the borrowed money, it is known as **simple interest**.

The money borrowed or lent is known as the **Principal**.

The sum of the Principal and the simple interest is known as the **amount**.

The rate at which interest is calculated is known as **rate** of **interest**.

Let P = Principal, T = Time in years, R = % rate of interest, then, the formula to calculate simple interest (S.I.) is given as:

$$\text{S.I.} = \frac{P \times T \times R}{100}$$

and the amount (A) at the end of the period is given as:

$$A = \text{Principal} + \text{Interest} = P + \text{S.I.}$$

> **Note:** Time should be in years. If not, divide it by 365 to convert it into years.

If time is in weeks, divide it by 52 to change it in years.

If time is in months, divide it by 12 to change it into years.

Example 12: *Calculate the simple interest on L$ 50000 for 4 years at the rate of 3% per annum. Also, find the total amount to be paid.*

Solution:

Here, P = L$ 50000, T = 4 years, R = 3% per annum

The simple interest is given by:

$$\text{S.I.} = \frac{P \times T \times R}{100} = \frac{50000 \times 4 \times 3}{100}$$

$$= \text{L\$ } 6000$$

Also, the total amount to be paid is given as:

$$A = P + S.I. = 5,0000 + 6000 = L\$ \ 5,6000.$$

Note: While calculating time, if specific dates are given, then, the day of deposit is not taken into consideration but day of withdrawal is considered.

Example 13: *Albert deposits L$ 30,000 into a bank account that pays a simple interest rate of 7.5% per annum. For how many years must he invest to generate L$ 45,000?*

Solution: Here

$$P = L\$ \ 30,000,\ r = 7.5\%,\ A = L\$ \ 45,000,\ T = ?$$

$\therefore$ Using A = P + S.I., we have

$$S.I. = A - P = 45,000 - 30,000 = L\$ \ 15,000$$

Also, $\quad S.I. = \dfrac{P \times T \times R}{100}$ gives

$$1500 = \frac{30000 \times T \times 7.5}{100} = 2250\ T$$

$$\Rightarrow \quad T = \frac{15000}{2250}$$

$$= 6.6 \text{ years}$$

i.e., he needs 6 years and 6 months to get the desired amount.

Example 14: *Daniel deposited a sum of L$ 75000 at the rate of 4% in the bank on August 5, 2021. On October 17, 2021, he withdrew the sum deposited along with the interest. Find the amount he got.*

Solution: Here, P = L$ 75000, R = 4% p.a.

To calculate T

From August 5 to August 31, we have 26 days (August 5 is not included but August 31, will be included)

There are 30 days in September and 17 days up to October 17, 2021.

$$\therefore \quad T = 26 + 30 + 17 = 73 \text{ days} = \frac{73}{365} \text{ years}$$

$\therefore$ The simple interest is given as:

$$\text{S.I.} = \frac{P \times T \times R}{100} = \frac{75000 \times 73 \times 4}{365 \times 100}$$

$$= \text{L\$ } 600$$

Daniel got total amount = P + S.I. = 75000 + 600

$$= \text{L\$ } 75600$$

Example 15: *Jerelyn invested an amount of L$ 13,9000 divided in two different schemes A and B at the simple interest rate of 14% per annum and 11% per annum respectively. Let the total amount of simple interest earned in 2 years be L$ 35080, what was the amount invested in schemes A and B?*

Solution: For scheme A: Let the amount invested = L$ x

R = 14%, T = 2 years

$\therefore$ The simple interest on scheme A is given as:

$$\text{S.I.} = \frac{P \times T \times R}{100} = \frac{x \times 2 \times 14}{100} = \frac{28x}{100} \quad \ldots(1)$$

For scheme B: The amount invested = L$ (139000 – x)

T = 2 years, R = 11% per annum

$\therefore$ The simple interest on scheme B is given as:

$$\text{S.I.} = \frac{P \times T \times R}{100} = \frac{(139000 - x) \times 2 \times 11}{100}$$

$$= \text{L\$ } \frac{11}{50}(1390000 - x) \quad \ldots(2)$$

Adding (1) and (2),

Total simple interest = 35080 (Given)

$$\frac{28x}{100} + \frac{11}{50}(139000 - x) = 35080$$

$$\Rightarrow \quad 28x + 22(139000 - x) = 3508000$$

$$\Rightarrow \quad 6x + 3058000 = 3508000$$

$$\Rightarrow \quad 6x = 3508000 - 3058000 = 450000$$

$$\Rightarrow \quad x = \frac{450000}{6} = 75000$$

∴ The amount invested in scheme A = L$ 75000

The amount invested in scheme B

= 139000 – 75000 = L$ 64000

EXERCISE 3.5

1. Find the simple interest on L$ 24000 at a rate of 7.5% per annum for 2 years.
2. Find the simple interest on L$ 5600 at 4½% per annum for 6 months.
3. If P = L$ 3000, R = 5% and T = 4 years, find S.I. and the amount at the end of the period.
4. Sarah deposited L$ 4500 at the bank at a rate of 5% per annum for 3 years. Find the amount she got at the end of the third year.
5. Find the principal on which the interest for 4 years at 5% per annum is L$ 1800.
6. The simple interest on L$ 7500 for 4 years is L$ 1050. What is the rate of interest per annum?
7. Find the time in which the interest on L$ 6000 at 5% is L$ 900.
8. Samuel borrowed a sum of money from a bank at an interest rate of 12%. After 1 year, he paid L$ 72800 to settle the loan and the interest. How much did he borrow from the bank?

Compound Interest

The interest calculated on the amount of the previous year is known as **compound interest** (C.I.) or **interest compounded**. In case of simple interest, the principal amount remains the same throughout the period of loan or investment.

But in case of compound interest, the interest earned at the end of each period becomes part of the principal for the following year and starts earning interest as well. We can calculate amount and compound interest (C.I.) by finding interest (simple interest) year by year or by using formula.

To calculated compound interest for a given period, without-using formula of compound interest, we use the method of **unitary method** which is based on calculating simple interest year by year.

Unitary Method for Calculating Compound Interest:

In **Unitary method**, we donot **require** any formula to calculate the compound interest for a given period. In this method, the period of compounding interest is taken as one year (or half year or quarter) and interest for each period is calculated by the following procedure.

Step I: Let P_1 = Principal value at the beginning of first period.
I_1 = Interest at the end of first period
A_1 = Amount at the end of first period, then,
$A_1 = P_1 + I_1$, where
I_1 = Rate of first period × P_1

Step II: Let P_2 = Principal value at the beginning of 2nd period = A_1
I_2 = Interest at the end of 2nd period
A_2 = Amount at the end of 2nd period, then,
$A_2 = P_2 + I_2$, where
I_2 = Rate of 2nd period × P_2

Step III: Let P_3 = Principal value at the beginning of 3rd period = A_2,
I_3 = Interest at the end of 3rd period
A_3 = Amount at the end of 3rd period, then,
$A_3 = P_3 + I_3$, where
I_3 = Rate of 3rd period × P_3 and so on.

Example 16: *Hassan deposited L$ 27000 in a bank which offers compound interest at the rate of 12% per annum.*

(a) *Find the compound interest, which he will get at the end of fourth year.*
(b) *Calculate his total amount at the end of fourth year.*
(c) *What will be his total compound interest.*

Solution: Here P_1 = Principal value at the beginning of first year = L$ 27000

I_1 = Interest at the end of first year
= 12% of L$ 27000

$$= \frac{12}{100} \times 27000 = \text{L\$ } 3240$$

$\therefore$ A_1 = Amount at the end of first year

$= 27000 + 3240 = \text{L\$ } 30240$

Now, P_2 = Principal value at the beginning of 2nd year = A_1 = L$ 30240

I_2 = Interest at the end of 2nd year

= 12% of L$ 30240

$$= \frac{12}{100} \times 30240 = \text{L\$ } 3628.80$$

$\therefore$ A_2 = Amount at the end of 2nd year

$= P_2 + I_2 = 30240 + 3628.80$

= L$ 33868.8

Again, $P_3 = A_2$ = L$ 33868.8

I_3 = 12% of L$ 33868.8

= L$ 4064.256

$\therefore$ $A_3 = P_3 + I_3$ = L$ 37933.056

Finally, $P_4 = A_3$ = L$ 37933.056

I_4 = 12% of L$ 37933.056

= L$ 4551.9667

$\therefore$ $A_4 = P_4 + I_4$ = L$ 42485.0227

(a) At the end of fourth year, the interest obtained by Hassan

= L$ 4551.9667 $\cong$ L$ 4552.

(b) The total amount at the end of fourth year

$= A_4$ = L$ 42485.0227 $\cong$ L$ 42485

(c) Now, total compound interest

$= 42485.0227 - 27000$

= L$ 15485.0227 $\cong$ L$ 15485

Note:

(i) If the period of calculation of compound interest is half yearly, then, the rate of interest will become $\frac{R}{2}\%$.

(ii) If the period of calculation of compound interest is quarterly, then the rate of interest will become $\frac{R}{4}\%$.

Example 17: *Bank of Liberia is offering an interest of 12% per annum compounded quarterly on a deposit of L$ 25000. How much interest one can get at the end of one year? Also, calculate the total amount at the end of one-year.*

Solution: Since the period of compounding interest is quarterly, therefore,

$$\text{Rate of interest} = \frac{12}{4}\% = 3\%$$

There are four periods in a year if the interest is compounded quarterly.

$\therefore$ P_1 = Principal value at the beginning of first period = L$ 25000

I_1 = Interest at the end of first period = 3% of L$ 25000 = L$ 750

A_1 = Amount at the end of first period = 25000 + 750 = L$ 25750

Further, P_2 = Principal value at the beginning of 2nd period = A_1 = L$ 25750

I_2 = Interest at the end of 2nd period = 3% of L$ 25750 = L$ 772.50

$\therefore$ $A_2 = P_2 + I_2$ = L$ 26522.50

Similarly, $P_3 = A_2$ = L$ 26522.50

I_3 = 3% of L$ 26522.50 = L$ 795.675

$\therefore$ $A_3 = P_3 + I_3$ = L$ 27318.175

Also, $P_4 = A_3$ = L$ 27318.175

I_4 = 3% of L$ 27318.175 = L$ 819.54525

$\therefore$ $A_4 = P_4 + I_4 = 28137.72025 \cong$ L$ 28138

$\therefore$ Required interest at the end of one year

$= I_1 + I_2 + I_3 + I_4$

$= 75 + 772.5 + 795.675 + 819.54525$

$= 3137.72025 \cong$ L$ 3138

Also, the total amount at the end of one year = A_4 = L$ 28138.

EXERCISE 3.6

1. A sum of L$ 20000 is borrowed by Eleanor for 2 years at an interest of 8% compounded annually.
   (i) Find the Compound Interest (C.I.) which she will pay at the end of 2 years.
   (ii) Find the amount she has to pay at the end of 2 years.
   (iii) What will be her total compound interest?
2. Felix deposited L$ 12000 in a bank at 12% compound interest per annum. Find his amount at the end of the third year and the total compound interest.
3. Find the amount at the end of 4 years, if L$ 5000 is invested at 5% per annum, at
   (i) Simple interest
   (ii) Compound interest

3.5. COMPOUND INTEREST FORMULAE

The Unitary Method which is used so far to calculate compound interest year by year becomes cumbersome when the years are many. To tackle any kind of situation, compound interest formula is a shorter or easier way of finding compound interest. Therefore, we can use this formula as a short-cut to calculate compound interest.

Compound Interest Formula

If P = Principal, n = No. of periods/years, R = Rate of interest per period/year, A = Amount after n periods/years, then, amount at the end of n periods/years is given by;

$$A = P\left(1+\frac{R}{100}\right)^n = P\,(1 + \text{rate in }\%)^n$$

Using above formula, we can calculate the compound interest as:

$$\text{Compound interest (C.I.)} = A - P = P\left(1+\frac{R}{100}\right)^n - P$$

Notes:

- If the interest is compounded half yearly ($\frac{1}{2}$ year), then we divide the rate by 2 and multiply the time by 2 before using the general formula for compound interest i.e., we have to take $\frac{R}{2}$% as a half year rate and also take the period/year as $2n$.
- If the interest is compounded quarterly ($\frac{1}{4}$ year), then we divide the rate by 4 and multiply the time by 4 before using the general formula for compound interest i.e., we have to take $\frac{R}{4}$% as a quarterly rate and also take the period/year as $4n$.
- Similarly, if we compound monthly, we have to take $\frac{R}{12}$% as a quarterly rate and also take the period/year as $12n$.

Applications of Compound Interest Formula

There are some situations where we could use the formula for calculation of amount in CI. Here are a few.

(i) Increase (or decrease) in population.

(ii) The growth of bacteria if the rate of growth is known.

(iii) The value of an item, if its price increases or decreases in the intermediate years.

Example 18: *An investments of L$ 25,000 earns interest of 9%, compounded annually. What will be the value of the investment at the end of 4 years?*

Solution: Given, investment = L$ 25000, rate of interest = 9% per annum, time = 4 years

$\therefore \quad P = \text{L\$ } 25000,\ r = 9,\ n = 4$

Using formula $\quad A = P\left(1+\frac{R}{100}\right)^n$, we have

$$\Rightarrow \quad A = 25000\left(1+\frac{9}{100}\right)^4 = 25000\left(\frac{109}{100}\right)^4$$

$$= 25000\ (1.09)^4$$

Taking logarithm (base 10), we get

$$\log_{10} A = \log_{10} 25000 + 4 \log_{10} (1.09)$$

$$= 4.397940009 + 4(0.037426497)$$

$$= 4.397940040009 + 0.149705991$$
$$= 4.547646001$$
$\Rightarrow$ $$A = \text{antilog}_{10}\,(4.547646001)$$
$$= 35289.54028 \cong 35290$$

$\therefore$ Value of investment after 4 years

$$= \text{L\$ } 35290$$

Example 19: *If L$ 17500 are invested at 5% interest per year for 2 years. Find*

(i) the interest compounded annually
(ii) the interest compounded half yearly
(iii) the difference between interests in part (i) and (ii)

Solution: (i) When interest compounded annually.

Given, Principal = L$ 17500, Rate of interest = 5% per annum, Time = 2 years

$\therefore$ $$P = \text{L\$ } 17500,\ r = 9,\ n = 2$$

Using formula $A = P\left(1 + \dfrac{R}{100}\right)^n$, we have

$$A = 17500\left(1 + \frac{5}{100}\right)^2$$
$$= 17500 \times \left(\frac{105}{100}\right)^2$$
$$= 17500 \times (1.05)^2$$
$$= 17500 \times (1.1025)$$
$$= \text{L\$ } 19293.75$$

$\therefore$ $$\text{Interest} = 19293.75 - 17500$$
$$= \text{L\$ } 1793.75$$

(ii) When interest compounded half yearly.

$\therefore$ $$P = \text{L\$ } 17500,\ R = \frac{5}{2} = 2.5,$$
$$n = 4 \text{ half years}$$

Using formula $A = P\left(1+\frac{\frac{R}{2}}{100}\right)^n$, we have

$$= 17500\left(1+\frac{5}{200}\right)^4$$

$$= 17500\left(\frac{205}{200}\right)^4$$

$$= 17500\ (1.025)^4 \qquad \ldots(1)$$

$$= 19316.73$$

$\therefore$ Interest $= A - I$

$= L\$\ 19316.73 - 17500$

$= L\$\ 1816.73$

(iii) Difference between interests

$= 1816.73 - 1793.75$

$= L\$\ 22.98 \cong L\$\ 23$

EXERCISE 3.7

1. If an amount L$ 8580 is invested at 10% per year interest, compounded every year for 5 years, what is the amount realized at the end of 5 years?

2. A sum of L$ 20000 is borrowed by Emine for 2 years at an interest of 8% compounded annually. Find the Compound Interest (C.I.) and the amount she has to pay at the end of 2 years.

3. Find the compound interest on $400 in 3 years at 4% interest per year. (Give answer nearest dollar)

4. Find the compound interest on L$ 75000 for 5 years at a rate 5% , compounded half yearly.

5. What amount is to be repaid on a loan of L$ 24000 for $1\frac{1}{2}$ years at 10% per annum, compounded half yearly.

6. Find how many years it will take the sum of L$ 35,000 to double when invested at 4% compounded half yearly?

3.6. DEPRECIATION

Depreciation means reduction of value due to use over a period of time and age of the item. The value of item decreases or depreciates by a percentage of the original value.

Items that lose their values over a period time like cars, bikes, TVs, radios, fridges, electronic machines and industrial machines are examples of depreciation.

If an item depreciates by x%, then, its new value is $(100 - x)$% of the original value. Also,

(i) New value $= \dfrac{100 - x}{100} \times$ Original value

(ii) Original value $= \dfrac{100}{100 - x} \times$ New value

(iii) Depreciation % $= \dfrac{\text{Reduction in value}}{\text{Original value}} \times 100$

> **Note:**
> If V_0 = value of a item at beginning, n = no. of periods, r = rate of depreciation of item per period, V_n = value of item at the end of n periods, then
>
> $$V_n = V_0\left(1 - \frac{r}{100}\right)^n.$$

Example 20: *A TV was bought at L$ 52500. Its value depreciated at a rate of 8% per annum. Find it new value*

(i) after one year. (ii) after two year.

Solution: Here, original value of TV, V_0 = L$ 52500, Rate of depreciation, $x = 8\%$

(i) New value of TV after one year

$$= \frac{100 - x}{100} \times \text{Original value}$$

$$= \frac{100 - 8}{100} \times \text{L\$}52500 = \frac{92}{100}\text{L\$}52500 = \text{L\$ } 48300$$

Alternatively,

We may directly get this as follows:

$\therefore$ New value of TV after one year,

$$V_1 = V_0\left(1-\frac{x}{100}\right) = \text{L\$}52500 \times \left(1-\frac{8}{100}\right)$$

$$= \text{L\$ } 52500 \times \frac{92}{100}$$

$$= \text{L\$ } 48300$$

(ii) New value of TV after two years,

$$V_2 = V_0\left(1-\frac{x}{100}\right)^2 = \text{L\$}52500 \times \left(1-\frac{8}{100}\right)^2$$

$$= \text{L\$ } 52500 \times \frac{92}{100} \times \frac{92}{100}$$

$$= \text{L\$ } 44436$$

Example 21: *The original price of a mobile phone was reduced by 17%. The new price of the mobile phone was L\$ 11620. Find the original price.*

Solution: Here x = 17%, new price = L\$ 11620

$$\therefore \quad \text{Original value} = \frac{100}{100-17} \times 11620$$

$$= \frac{100}{83} \times 11620$$

$$= \text{L\$ } 14000$$

Example 22: *Henry brought a radio for L\$ 7500. After one year, its cost depreciates by 25% and he sold it for L\$ 6000. Find his profit or loss percent.*

Solution: Here x = 25% and original cost of the radio = L\$ 7500. Therefore, after one year,

$$\text{New value} = \frac{100-25}{100} \times 7500 = \frac{75}{100} \times 7500 = \text{L\$ } 5625$$

But he sold the radio for L$ 6000. Therefore,

$$\text{Profit} = 6000 - 5625 = \text{L\$ } 375$$

$$\therefore \text{ Profit percentage} = \frac{\text{Profit}}{\text{New value}} \times 100$$

$$= \frac{375}{5625} \times 100 = 6.66\%$$

Example 23: *The price of a motor bike depreciates by 2% of its value at the beginning of each year. Find the sale value of the motor bike after 3 years if its present sale value is L$ 200000.*

Solution: Let P = Present sale value of the motor bike = L$ 200000

Rate of depreciation, $r = 2\%$, $n = 3$ years

Let S = Sale value after 3 years.

Then,
$$S = P\left(1 - \frac{r}{100}\right)^n$$

$$= 200000\left(1 - \frac{2}{100}\right)^3$$

$$= 200000\left(\frac{98}{100}\right)^3$$

Taking logarithm (base 10) both sides,

$$\log_{10} S = \log_{10} 200000 + \log_{10}\left(\frac{98}{100}\right)^3$$

$$= \log_{10} 200000 + 3(\log_{10} 98 - \log_{10} 100)$$
$$= 5.30102 - 3\,(1.99122 - 2)$$
$$= 5.30102 - 3\,(0.0088)$$
$$= 5.30102 - 0.0264 = 5.27462$$

Taking antilog, both sides

$$S = \text{antilog}_{10}\,(5.27462)$$
$$= 188200.165 \equiv 188200$$

i.e., the sale value of the motor bike after 3 years = L$ 188200.

Example 24: *A machine is depreciated at the rate of 10% on reducing balance. The original cost was L$ 10000 and the ultimate scrap value was L$ 3750. Find the effective life of the machine?*

Solution: Rate of depreciation = 10%

i.e., $r = 10$

V_0 = Value of machine in the beginning = L$ 10,000.

V_n = Value of machine at the end of n years = L$ 3750.

n = Effective life of the machine.

Using $V_n = V_0 \left(1 - \frac{r}{100}\right)^n$, we have

$$3750 = 10000\,(1 - 0.1)^n = 10000\,(0.9)^n$$

Taking logarithm (base 10), both sides, we get

$$\log_{10} 3750 = \log_{10} 10^4 + n \log_{10} (0.9)$$

$$\Rightarrow \quad 3.57403 = 4 + n(-0.045757)$$

$$= 4 - (0.045757)\, n$$

$$\Rightarrow \quad (0.045757)\, n = 4 - 3.7403 = 0.42597$$

$$\Rightarrow \quad n = \frac{0.42597}{0.045757} = 9.3093 = 9 \text{ years.}$$

Example 25: *The cost price of a new car is L$ 3.7 × 10^6. The insurance company calculates its price at any subsequent time according to the rule that the price depreciates at the rate of 5% a year during first two years and at the rate of 10% a year there after. What will be the price of the car after 5 years?*

Solution: Cost of new car = L$ 3.7 × 10^6, Rates of depreciation = 5% p.a. for first 2 years and 10% p.a. there after, time = 5 years.

$\therefore \quad V_0 = 3.7 \times 10^6,\ r_1 = 5\%,$

$r_2 = 10\%,\ n_1 = 2,\ n_2 = 3\ (= 5 - 2).$

Using $V_n = V_0 \left(1 - \frac{r_1}{100}\right)^{n_1} \left(1 - \frac{r_2}{100}\right)^{n_2}$, We have (AL = antilog_{10})

$$V_5 = 3.7 \times 10^6 \left(1 - \frac{5}{100}\right)^2 \left(1 - \frac{10}{100}\right)^3$$

$$= 3.7 \times 10^6 (0.95)^2 (0.9)^3$$

$$= \text{AL} [\log_{10} [3.7 \times 10^6 (0.95)^2 (0.9)^3]$$

$$= \text{AL} [\log_{10} 3.7 + 6 \log_{10} 10 + 2 \log_{10} 0.95 + 3 \log_{10} 0.9]$$

$$= \text{AL} [0.5682 + 6(1) + 2(\bar{1}.9777) + 3(\bar{1}.9542)]$$

$$= \text{AL} [0.5682 + 6 + 2(-1) + 2(0.9777) + 3(-1) + 3(0.9542)]$$

$$= \text{AL} [6.3862] = 2433000$$

$$= 2.433 \times 10^6.$$

$\therefore$ Price of car after 5 years

$$= \text{L\$ } 2.433 \times 10^6.$$

EXERCISE 3.8

1. A TV was bought at a price of L\$ 42,000. After one year the value of the TV was depreciated by 5%. Find the value of the TV after one year.
2. The original price of an electronic item was reduced by 24%. The new price of the article was L\$ 22800. What was the original price?
3. Ousmane bought a radio for L\$ 10000. After one year the cost depreciates by 25%.
   (i) What was the value of the radio after one year?
   (ii) If he sold the radio for L\$ 8500, calculate his profit or loss percent.
4. The value of a printing machine depreciated each year by 5% of its value at the beginning of that year. If the value of a new machine is L\$ 80000, find its value at the end of the third year.
5. The value of a car depreciates by 10% of its value in the first year. In subsequent years, the depreciation is 15% of its value at the beginning of the year. The value of the car when new is \$4500. Find its value when it is 3 years old.

3.7. HIRE PURCHASE

The term hire purchase means a way of buying expensive goods such as T.V., Friz, Car, Flat etc. Using hire purchase, one pays a small amount in the beginning and the remaining amount to be paid by monthly or yearly installments.

Note:
Buying of goods on hire purchase usually costs more to calculate the total hire purchase price, we add the deposit and the total of all of the installments.

Example 26: *A pressure cooker can be bought on hire purchased price by paying a deposit of 35% and 36 monthly payments of L$ 114. The cost of pressure cooker in cash price is L$ 4600. Find the hire purchase price of the pressure cooker.*

Solution: Deposit amount

$$= 35\% \text{ of L\$ } 4600 = \frac{35}{100} \times 4600 = \text{L\$ } 1610$$

One installment cost = L$ 114

$\Rightarrow$ 36 installments cost = 36 × 114 = L$ 4104

$\therefore$ Hire purchase price = Deposit amount + Total installments cost

= 1610 + 4104 = L$ 571.40

Example 27: *Electroland Ghana Limited (EGL) launched a new T.V. which costs L$ 32850 on cash payment. It is available on hire purchase price by paying a deposit of 15% followed by 12 installments of L$ 2677.50. Find the total hire purchase price and the extra money that you would pay (over the cash price) using hire purchase.*

Solution: Deposit amount = 15% of L$ 32850

$$= \frac{15}{100} \times 32850 = \text{L\$ } 4927.50$$

1 installment cost = L$ 2677.50

$\therefore$ 12 installments cost = 2677.50 × 12 = L$ 32130

$\therefore$ Hire purchase price = deposit amount

+ 12 installments cost

$= 4927.50 + 32130 = \text{L\$ } 37057.50$

Given, cash price = L$ 32850

$\therefore$ Extra amount to be paid = Hire purchase price – Cash price

$= 37057.50 - 32850 = \text{L\$ } 4207.50$

Example 28: *The cash price of a computer in Monrovia is $ 550. The hire purchase price is $ 625. If you pay a deposit of 15% followed by 20 equal monthly installments, find how much you pay (in L$) per month. The rate of exchange is L$1 = $ 0.0065*

Solution: Deposit amount = 15 % of $ 550

$$= \frac{15}{100} \times 550 = \$\ 82.50$$

$\therefore$ 20 installments cost = Hire purchase price – Deposit amount

$= 625 - 82.50 = \$\ 542.50$

$$\Rightarrow \quad 1 \text{ installment cost} = \frac{542.50}{20} = \$\ 27.125$$

We need to find the cost of one installment in Ghana currency.

Given L$ 1 = $ 0.0065

$$\Rightarrow \quad \$\ 1 = \text{L\$ } \frac{1}{0.0065}$$

$= \text{L\$ } 153.8462$

$\therefore$ $ 27.125 = 153.88462 × 27.125

$= \text{L\$ } 4173.0782$

Thus, one has to pay = L$ 4173 per month.

Example 29: *A DVD player is priced at L$ 4800 in different shops A and B which offer different hire purchase terms. Shop A requires 20% deposit and 12 monthly installments of L$ 399. Shop B requires 30% deposit and 12 monthly installments of L$ 352.50. Which shop has the better deal?*

Solution: For shop A:

Deposit amount = 20% of 4800

$$= \frac{20}{100} \times 4800 = \text{L\$ } 960$$

1 installment cost = L$ 399

$\therefore$ 12 installments cost $= 12 \times 399$

$= \text{L\$ } 4788$

$\therefore$ Hire purchase price = Deposit amount + Total installment cost

$= 960 + 4788 = \text{L\$ } 5748$

For shop B: Deposit amount = 30% of 4800

$= \frac{30}{100} \times 4800 = \text{L\$ } 1440$

1 installments cost = L$ 352.50

$\therefore$ 12 installments cost $= 12 \times 352.50$

$= \text{L\$ } 4230$

$\therefore$ Hire purchase price = Deposit amount + Total installments cost

$= 1440 + 4230 = \text{L\$ } 5670$

$\therefore$ The shop B has the better deal.

EXERCISE 3.9

1. A car can be bought on hire purchase price by paying a deposit of 20% and 48 monthly payments of L$ 7500. The cost of car in cash price is L$ 400000. Find the hire purchase price of the car.
2. A DVD player costs L$ 12000 cash. It is available on hire purchase price by paying a deposit of 35% followed by 12 installments of L$ 725, Find the extra paid by hire purchase.
3. The cash price of a bike is $ 700. The hire purchase price is $ 720, If the deposit is 10% followed by 10 equal monthly installments, find the amount you pay each month.
4. The cash price of an electric cooker is L$ 7800. The hire purchase terms are: Deposit 24% of the cash price and 24 equal monthly installments of L$ 320.75. Calculate the total hire purchase price of the cooker. Also, find the extra paid by hire purchase.

MULTIPLE CHOICE QUESTIONS

1. The value of $3\sqrt{500} - 2\sqrt{125}$, when simplified, is

(a) $2\sqrt{5}$ (b) $4\sqrt{5}$

(c) $10\sqrt{5}$ (d) $20\sqrt{5}$

2. The value of $(4-\sqrt{5})(3+\sqrt{5})$, when simplified, is

(a) $7-\sqrt{5}$ (b) $7+\sqrt{5}$

(c) $3-\sqrt{5}$ (d) $4+\sqrt{5}$

3. The value of $(14+3\sqrt{7})\left(2-\frac{3}{\sqrt{7}}\right)$, when simplified, is

(a) 7 (b) 13

(c) 19 (d) 23

4. The number $\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{2}}$ when expressed as a surd is

(a) $\frac{2}{3}\sqrt{2}$ (b) $\frac{1}{3}\sqrt{2}$

(c) $\frac{2}{3}\sqrt{3}$ (d) $\frac{1}{3}\sqrt{3}$

5. If $\sqrt{7}$ = 2.646, the value of $\sqrt{0.0007}$ is

(a) 0.02646 (b) 0.2664

(c) 0.2666 (d) 0.002646

6. If $\sqrt{5}$ = 2.236, then, the value of $2\sqrt{5}(6-2\sqrt{5})$, when simplified, is

(a) 6.833 (b) 6.832

(c) 6.823 (d) 6.822

7. Let * be a binary operation defined on reals such that $a * b = a^2 + 2ab + b^2$, then, value of $\sqrt{3} * \sqrt{12}$ is

(a) 72 (b) 27

(c) 17 (d) none of these

8. If $\frac{\sqrt{50}\times\sqrt{45}}{\sqrt{200}\times\sqrt{75}} = p\sqrt{q}$, where p is any rational number (non-zero) and q is a +ve integer, then,

(a) p = 15, q = 0.1 (b) p = 0.1, q = 15

(c) both (a) and (b) (d) none of these

9. If $(2-x\sqrt{3})(3+4\sqrt{3})=-18+2\sqrt{3}$ then, the value of x is

(a) 6 (b) – 6

(c) 2 (d) – 2

10. If $p+q\sqrt{5}=(1-\sqrt{5})\left(\frac{1}{5}+\sqrt{5}\right)$ and $p=-\frac{24}{5}$, then, the value of q is

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$

(c) $\frac{2}{5}$ (d) $\frac{1}{5}$

11. Let A denotes the amount invested, P is the principal sum, R is the % rate of interest and n is the number of period, then, the formula to calculate compound interest is

(a) A – P where $A = P\left(1+\frac{R}{100}\right)^n$ (b) P – A where $A = P\left(1+\frac{R}{100}\right)^n$

(c) A – P where $P = A\left(1+\frac{R}{100}\right)^n$ (d) P – A where $P = A\left(1+\frac{R}{100}\right)^n$

12. Mohammed deposited L$ 9500 in a bank at 11% compound interest per annum. The compound interest at the end of 2nd year is

(a) L$ 1150 (b) L$ 1160

(c) L$ 1170 (d) L$ 1180

13. William invested a sum of L$ 50000 in a bank at 5%. Compounded half yearly for 5 years. The interest hat William will get is (approx)

(a) L$ 12000 (b) L$ 14000

(c) L$ 13000 (d) L$ 16000

14. The original price of an item was reduced by 15%. The new price of the item is L$ 1020. The original price is

(a) L$ 1400 (b) L$ 1250

(c) L$ 1300 (d) L$ 1200

15. A printing machine is to be sold for L$ 5500 or by monthly installments with a deposite of L$ 2,492 along with a monthly payment of L$ 752. The payment will be completed in

(a) 4 months (b) 3 months

(c) 5 months (d) none of these